Activity 4 Combinations

1.

a) ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE

b) 3! = 6

c) 5×4×3=60

d) $\frac{60}{6} = 10$. Each group is represented in six photographs so the number of groups is the number of permutations divided by the number of ways the group can be arranged.

e) $10 = \frac{5 \times 4 \times 3}{3!} = \frac{5!}{2!3!}$

2.

a) ABC, ABD, ABE, ABF, ABG, ACD, ACE, ACF, ACG, ADE, ADF, ADG, AEF, AEG, AFG,
BCD, BCE, BCF, BCG, BDE, BDF, BDG, BEF, BEG, BFG
CDE, CDF, CDG, CEF, CEG, CFG
DEF, DEG, DFG, EFG.

b) 3!=6

c) 7×6×5=210

d) $\frac{7 \times 6 \times 5}{3!} = \frac{7!}{4!3!} = 35$

e) 35, for every group of three that have a ride there is a group of four left behind, i.e. it is the same.

3. a) to d) are each 10
e) to h) are each 35.

4. $C_r^n = C_{n-r}^n$

$$C_r^n = \frac{P_r^n}{r!} = \frac{n!}{(n-r)!r!}$$

5. Pascal's triangle can be written as Combinations. i.e.

$$\begin{array}{ccccccccc} & & & C_0^1 & & C_1^1 & & & \\ & & C_0^2 & & C_1^2 & & C_2^2 & & \\ & C_0^3 & & C_1^3 & & C_2^3 & & C_3^3 & \\ C_0^4 & & C_1^4 & & C_2^4 & & C_3^4 & & C_4^4 \end{array}$$

Etc

6. All statements are true.

a) It is the same whether selecting two to go or selecting two to leave out.

b)

$$\text{LHS} = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$\text{RHS} = \binom{n}{n-r}$$

$$= \frac{n!}{(\text{n}-(n-r))!(n-r)!}$$

$$= \frac{n!}{r!(n-r)!}$$

$$= \text{LHS}$$

c) Both sides equal

d) Add two adjacent elements in a row of Pascal's triangle together to get the corresponding element in the next row.

OR

$$\text{LHS} = \binom{n}{r} + \binom{n}{r+1}$$

$$= \frac{n!}{(n-r)!r!} + \frac{n!}{(n-(r+1))!(r+1)!}$$

$$= \frac{n!}{(n-r)!r!} + \frac{n!}{(n-r-1)!(r+1)!}$$

$$= \frac{n!(r+1)}{(n-r)!(r+1)!} + \frac{n!(n-r)}{(n-r)!(r+1)!}$$

$$= \frac{n!(r+1)+n!(n-r)}{(n-r)!(r+1)!}$$

$$= \frac{n!(r+1+n-r)}{(n-r)!(r+1)!}$$

$$= \frac{n!(n+1)}{(n-r)!(r+1)!}$$

$$= \frac{(n+1)!}{(n-r)!(r+1)!}$$

$$= \binom{n+1}{r+1} = \text{RHS}$$